Vorlesungsergänzung zur Ingenieurmathematik

On frequency characteristics, characteristic polynomials and filter realization

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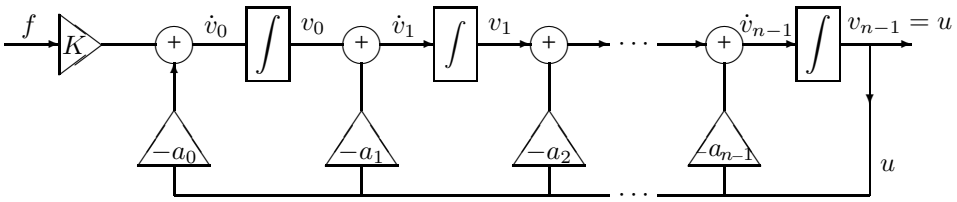
A Butterworth lowpass filter and many other filter types of order n can be described by the following signal flow diagram.



The diagram is the result of the fact, that the filter is represented by the linear differential equation

$$u^{(n)} + a_{n-1}u^{(n-1)} + \ldots + a_1\dot{u} + a_0 u = Kf,$$

which gives the input-output-relation for an input voltage f and the corresponding output voltage u. K is the dc-gain of the filter. (In the equation $u^{(k)}$ means the k-th derivative of a function u. A solution u of the equation for a given input voltage f is then the system output voltage.)
The fact, that the diagram and the given differential equation represent the same linear transfer system, is worked out in lectures on linear differential equations and linear transfer systems in the second semester.
The *frequency characteristics* $H(\omega)$ of the filter is then given by the rational function

$$H(\omega) = \frac{K}{P(j\omega)},$$

where $j^2 = -1$ and the polynomial P of degree n is

$$P(z) = z^n + a_{n-1}z^{n-1} + \ldots + a_0.$$

The polynomial P is called the *characteristic polynomial* of the differential equation. Thus, the coefficients of that polynomial P, in other words its zeroes or equivalently the poles of H determine the transfer properties of the filter.
For a specific Butterworth frequency characteristics please see the Maple document, where this page is linked to
(http://www.ohm-hochschule.de/aw/profs/brigola/Maple_html_export/Zu_Butterworth-Filtern.html).
For the existence of the frequency characteristics with the representation $H(\omega) = K/P(j\omega)$ it is a necessary condition, that the zeroes of P have negative real parts.
Then the signal diagram above can be realized by operational amplifiers, resistances and capacities building integrators, proportional amplifiers and voltage adders.
For a realization example please see the link to

http://www.ece.uic.edu/∼jmorisak/blpf.html